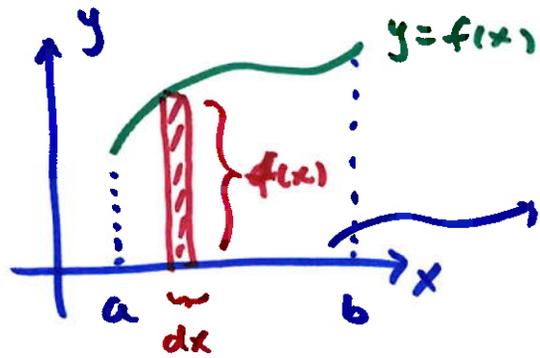


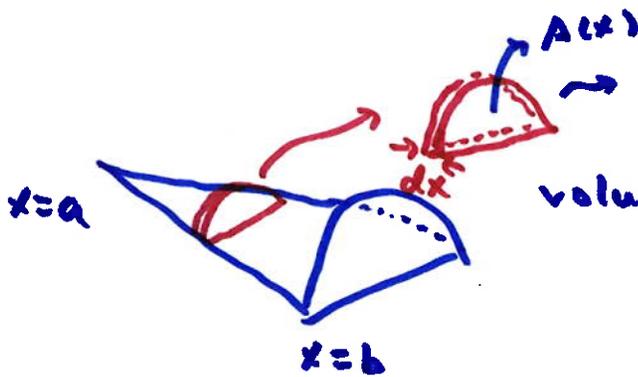
## 6.3 Volumes by Slicing



$$\text{area} = \int_a^b f(x) dx$$

thin rectangle area at  $x$

accumulates all from  $x=a$  to  $x=b$



volume of this is area at  $x$  times thickness  $dx$

volume of this?

$$\text{volume} = \int_a^b A(x) dx$$

↓  
area at  $x$

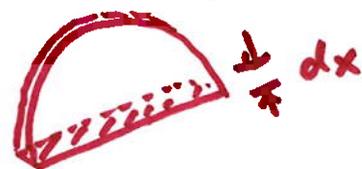
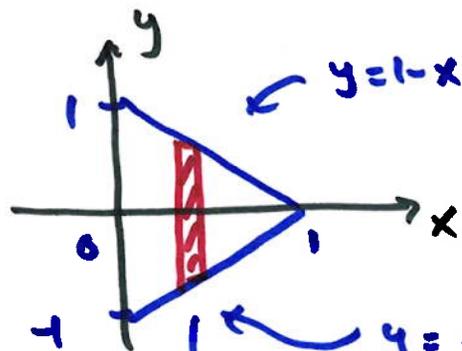
thickness of slice

example

the base of the object is region bounded by  $y=1-x$

and  $y=-1+x$  from  $x=0$  to  $x=1$

each slice perpendicular to  $xy$ -plane is a semicircle



↳ thickness is  $dx$

half a circle : area =  $\frac{1}{2} \pi (\text{radius})^2$

the shaded bar is the diameter of this semicircle

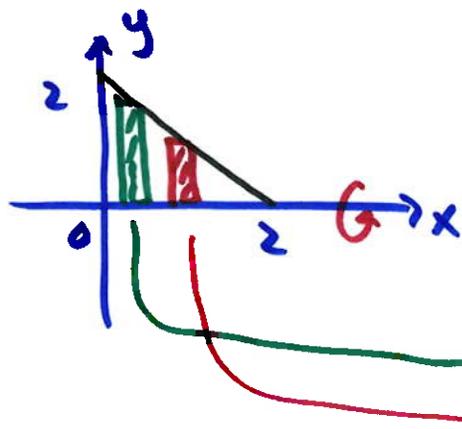
diameter = length of shaded bar =  $(1-x) - (-1+x) = 2-2x$

radius =  $1-x$

the volume of each slice =  $\frac{1}{2} \pi (1-x)^2 dx$  volume of

stack all possible slices :  $\int_0^1 \frac{1}{2} \pi (1-x)^2 dx = \dots = \boxed{\frac{\pi}{6}}$

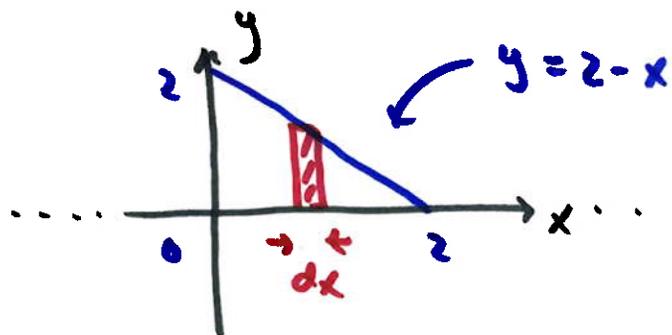
## Volume of revolution



revolve this region about  $x$ -axis  
the triangle sweeps out a cone

volume?

find the volume of each disk at some  $x$ , then integrate to accumulate



↓  
radius = height of bar =  $2 - x$

↑  
 $y = 0$



$$\text{volume} = (\text{area}) (\text{thickness}) = \pi (\text{radius})^2 dx$$

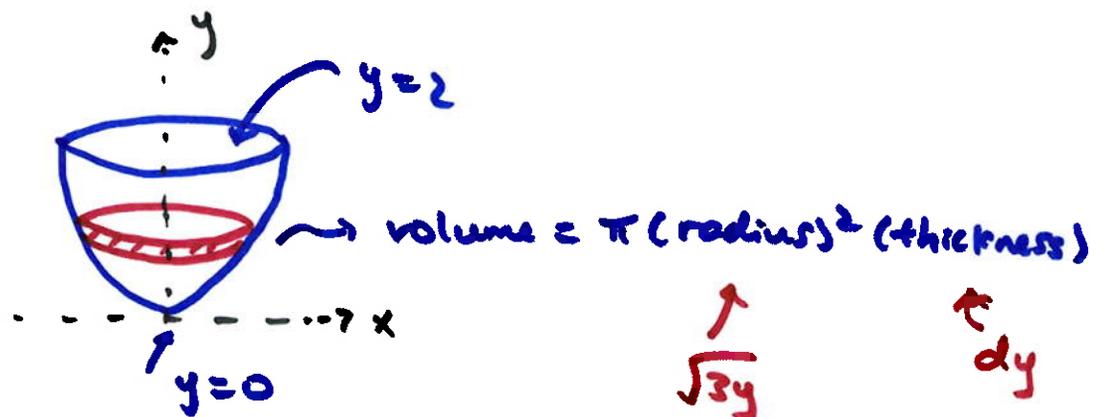
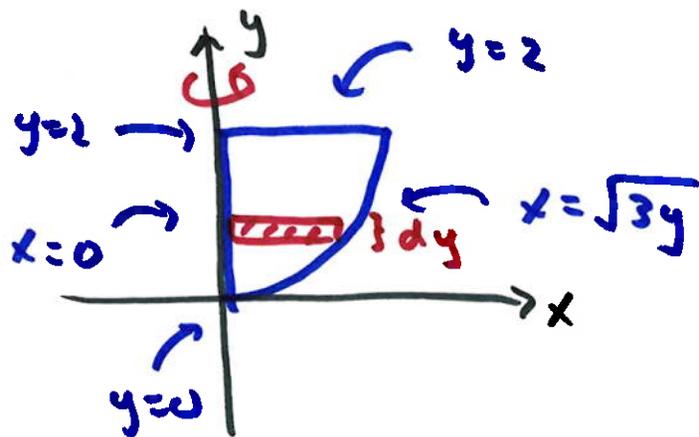
$$= \pi (2-x)^2 dx$$

this is one slice starting at  $x=0$ , ending at  $x=2$

integrate to accumulate:

$$\int_0^2 \pi (2-x)^2 dx = \dots = \boxed{\frac{8\pi}{3}}$$

example volume of solid obtained by revolving the region bounded by  $x = \sqrt{3y}$ ,  $x=0$ ,  $y=2$ , about the  $y$ -axis



all these  $y$ 's and  $dy$   $\rightarrow$  integrate in terms of  $y$

volume of one slice =  $\pi (\sqrt{3y})^2 dy$

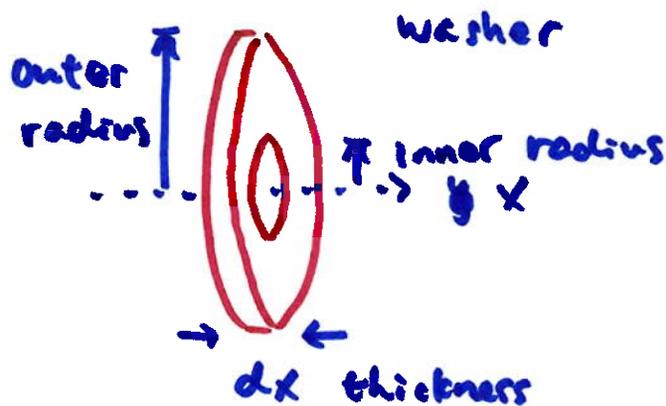
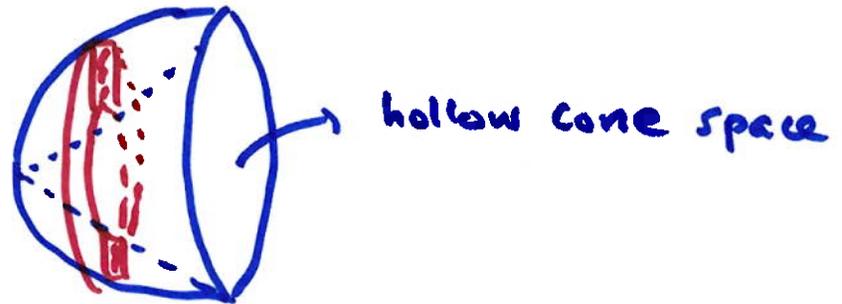
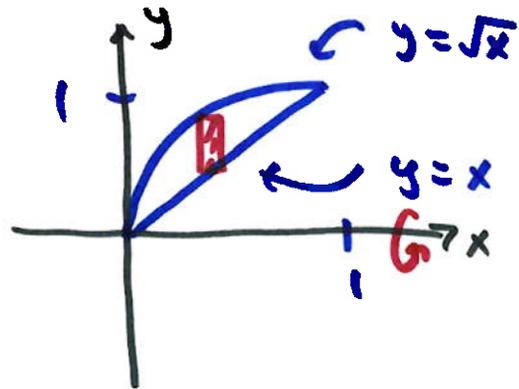
accumulate from  $y=0$  to  $y=2$

$$\int_0^2 \pi (\sqrt{3y})^2 dy = \int_0^2 3\pi y dy = 3\pi \frac{y^2}{2} \Big|_0^2 = \boxed{6\pi}$$

this method, accumulate disks (or washers) is called the Disk / Washer Method

example

Solid obtained by revolving the region bounded by  $y = \sqrt{x}$ ,  $y = x$ , about  $x$ -axis

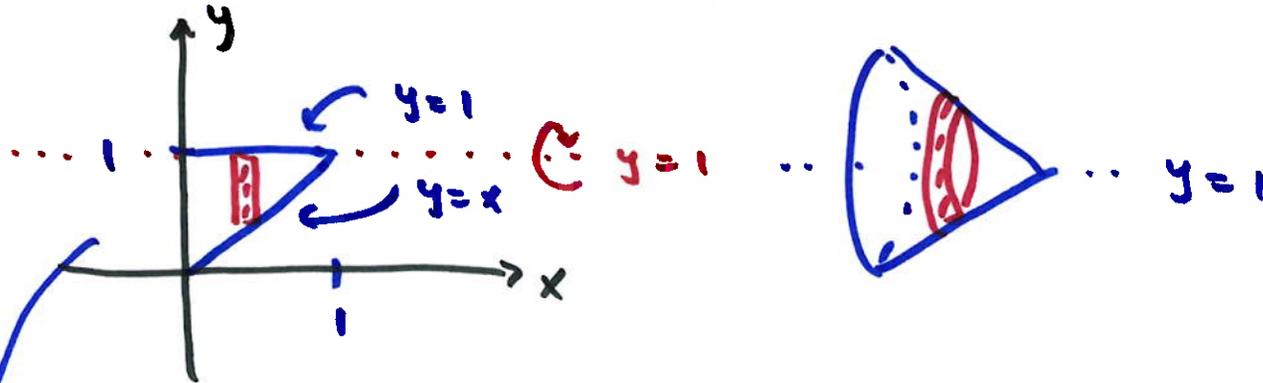


$$\begin{aligned} \text{area} &= \text{area of big} - \text{area of small} \\ &= \pi (\text{outer})^2 - \pi (\text{inner})^2 \end{aligned}$$

$$\begin{aligned} \text{volume of washer} &= \left[ \pi (\text{outer})^2 - \pi (\text{inner})^2 \right] dx = (\pi(x) - \pi(x^2)) dx \\ \text{start: } x &= 0 & \swarrow \sqrt{x} & \nearrow x \\ \text{end: } x &= 1 \end{aligned}$$

$$\text{volume of whole thing: } \int_0^1 [\pi(x) - \pi(x^2)] dx = \dots = \boxed{\frac{\pi}{6}}$$

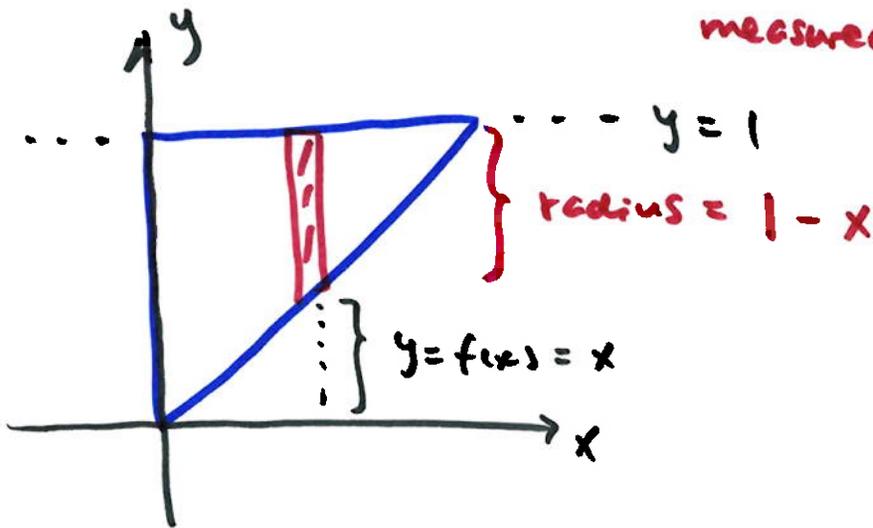
example region is bounded by  $y=1$ ,  $y=x$ ,  $x=0$   
 revolved around  $y=1$



disk volume =  $\pi$  (radius)<sup>2</sup> · thickness

here,  $dx$

measured from axis of revolution



$$\int_0^1 \pi (1-x)^2 dx$$

THE HELEN BASS WILLIAMS  
ACADEMIC SUCCESS CENTER PRESENTS

# TEST DRIVE: THE ELLIOTT HALL EXAM EXPERIENCE

WEDNESDAY, SEPTEMBER 6 • 7:00 PM

## COURSES OFFERED:

MA 165  
MA 261  
MGMT 200  
ECON 251  
PHYS 172

MA 162  
↙

## REGISTRATION IS REQUIRED

Test Drive is an opportunity to take a timed, mock exam to evaluate your content knowledge while experiencing the large, evening, and unfamiliar exam setting.

## SCAN FOR DETAILS & REGISTRATION



### Additional Information:

- You may only select one course option even if you are enrolled in multiple offerings
- You must be enrolled in the course you select
- DRC exam accommodations are available though this event
- Check in will run from 7pm to 7:45pm
  - Arrive during this time to receive your exam, find a seat, and settle before the start of the exam.



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**ON YOUR MARK, GET SET, GO!**